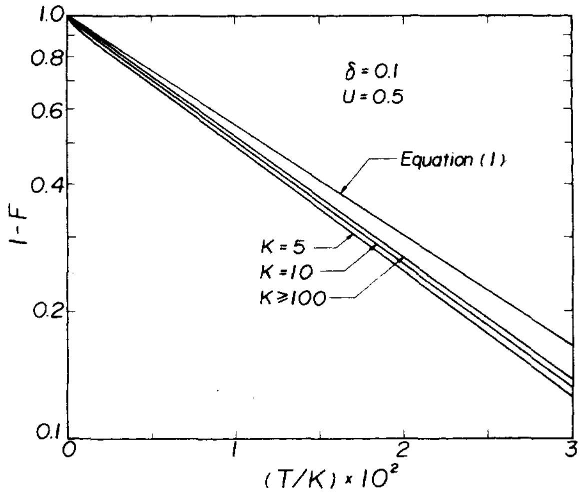

Figure 2. Plots of log(1 − F) vs. T/K with K as parameter for $\delta = 0.1$ and $U = 0.5$.

Figure 2 indicates the relationships between $\log(1 - F)$ vs. T/K for $\delta = 0.1$ and U - 0.5 with K as parameter. It can be seen that the exchange rate predicted from Eq. 9 is always faster than that predicted from Eq. 1 under the conditions of $U = 0.5$, $\delta = 0.1$ and any K values. The discrepancy is about 10% as $K \geq 100$, and is more obvious as the values of K decrease from 100.

Therefore, when the exchange rate is controlled by film diffusion in a finite bath, the exchange rate predicted from Eq. 9 is faster than that predicted from Eq. 1 under large δ and small K.

ACKNOWLEDGMENT

The author wishes to express his thanks to Prof. T. Vermeulen of the Department of Chemical Engineering, University of California at Berkeley, for his valuable suggestions and discussions.

NOTATION

a = radius of the resin particle, cm
b = distance from the center of a spherical resin particle to the outer boundary of the liquid film, cm
C = concentration of radioactive isotope in the liquid film, mol/cm^3
C' = concentration of radioactive isotope in the bulk solution, mol/cm^3
$\overline{C}$ = concentration of radioactive isotope in the resin particle, mol/cm^3
D = diffusion coefficient in the solution, cm^2/s
F = fractional attainment of equilibrium
gn = non-zero roots of Eq. 10
K = distribution coefficient
M = total amount of radioactive isotope transferred into the resin particles, mol
N = number of resin particles in the solution
r = distance from the center of a spherical resin particle, cm
t = time, s
T = Dt/a^2
U = final fractional uptake of radioactive isotope by the resin particles
V = volume of the solution, cm^3

Greek Letters

α = $3V/4\pi a^3NK$
δ = $(b - a)/a$

LITERATURE CITED

Boyd, G. E., A. W. Adamson, and L. S. Myers, Jr., "The Exchange Adsorption of Ions from Aqueous Solution by Organic Zeolites. II. Kinetics," *J. Am. Chem. Soc.*, **69**, 2836 (1947).
Helfferich, F., "Ion Exchange," p. 264, McGraw-Hill, New York (1962).
Huang, T. C., and F. N. Tsai, "Kinetic Parameters of Isotopic Exchange Reaction of Finite Bath," *Canadian J. Chem. Eng.*, **55**, 301 (1977).
Span, J., "Self-Diffusion of Iodide Ions into Ion-Exchange Resin Dowex 1 × 8," *J. Chem. Phys.*, **52**, 3097 (1970).

Manuscript received February 9, 1981; revision received June 30, and accepted August 7, 1981.

Conjugate Unsteady Heat Transfer from a Droplet in Creeping Flow

S. R. K. IYENGAR

Mechanical Engineering Department
Mosul University,
Mosul, Iraq
and
Department of Mathematics
Indian Institute of Technology
New Delhi-110016, India

Recently, in *AIChE Journal*, Abramzon and Borde (1980) have numerically solved the problem of unsteady heat transfer from a droplet in creeping flow and presented many discussions based on the computed results. Herein, we show that the finite-difference equations used in the above mentioned paper are unstable in part of the region considered.

The problem is to study the unsteady heat transfer from a spherical droplet moving slowly into an unbounded volume of another immiscible liquid. The Reynolds number is taken to be small (Re ≤ 1) and the flow fields are steady. At time $t = 0$, the temperature of the droplet and of the continuous phase are uniform and are equal to T_0 and T_∞ respectively. The physical properties

---

0001-1541-82-5661-0700-$2.00.

of the material within the droplet and in the surrounding medium are independent of the temperature and the thermal properties of the dispersed and continuous phases are identical. The nondimensional stream functions of the flow inside and outside the liquid sphere are given by the Hadamard-Rybezynski equations

$$\psi_1 = -\frac{r^2(1-r^2)}{4(1+K)}\sin^2\theta, \; (r \leqslant 1) \tag{1}$$

$$\psi_2 = 0.25\left(2r^2 - \frac{2+3K}{1+K}\cdot r + \frac{K}{1+K}\cdot\frac{1}{r}\right)\sin^2\theta, \; (r \geqslant 1). \tag{2}$$

Mathematically, the above problem finally reduces to finding the solution of the equations

$$\frac{\partial W}{\partial \tau} + \frac{\mathrm{Pe}}{2r^2\sin\theta}\left[-\frac{\partial\psi}{\partial\theta}\left(\frac{\partial W}{\partial r} - \frac{W}{r}\right) + \frac{\partial\psi}{\partial r}\frac{\partial W}{\partial\theta}\right] = \frac{\partial^2 W}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 W}{\partial\theta^2} + \frac{\cot\theta}{r^2}\frac{\partial W}{\partial\theta} \tag{3}$$

for the inner region $r \leqslant 1$, and

$$e^{2x}\frac{\partial W}{\partial \tau} + \frac{\mathrm{Pe}}{2e^x\sin\theta}\left[-\frac{\partial\psi}{\partial\theta}\left(\frac{\partial W}{\partial x} - W\right) + \frac{\partial\psi}{\partial x}\cdot\frac{\partial W}{\partial\theta}\right] = \frac{\partial^2 W}{\partial x^2} - \frac{\partial W}{\partial x} + \frac{\partial^2 W}{\partial\theta^2} + \cot\theta\cdot\frac{\partial W}{\partial\theta} \tag{4}$$

for the external region $r \geq 1$ (Eqs. 29 and 30 of the reference). Here $W = rZ$, and Z is the dimensionless temperature.

The initial and boundary conditions are

$$\begin{aligned} W(r,\theta,0) &= r, 0 \leqslant r < 1 \\ &= 0.5, r = 1 \\ &= 0, r > 1 \\ W(0,\theta,\tau) &= W(\infty,\theta,\tau) = 0 \end{aligned}$$

$$\frac{\partial W}{\partial\theta}(r,0,\tau) = \frac{\partial W}{\partial\theta}(r,\pi,\tau) = 0, \; 0 < r < \infty.$$

Write

$$f(r,\theta) = \frac{\mathrm{Pe}}{2r^2\sin\theta}\cdot\frac{\partial\psi}{\partial\theta} = -\frac{\mathrm{Pe}(1-r^2)}{4(1+K)}\cos\theta, \; r \leqslant 1$$

and

$$g(r,\theta) = \frac{1}{r^2}\left(\cot\theta - \frac{\mathrm{Pe}}{2\sin\theta}\frac{\partial\psi}{\partial r}\right) = \frac{1}{r^2}\left(\cot\theta + \frac{\mathrm{Pe}\,r(1-2r^2)}{4(1+K)}\sin\theta\right), \; r \leqslant 1.$$

Notice the property that $f(r,\theta) > 0$ for $\pi/2 < \theta < \pi$ and $f(r,\theta) < 0$ for $0 \leqslant \theta < \pi/2$. The sign of $g(r,\theta)$ is not of importance. Equation 3 for $r \leqslant 1$ can now be written as

$$\frac{\partial W}{\partial\tau} = \frac{\partial^2 W}{\partial r^2} + f(r,\theta)\left(\frac{\partial W}{\partial r} - \frac{W}{r}\right) + \frac{1}{r^2}\frac{\partial^2 W}{\partial\theta^2} + g(r,\theta)\frac{\partial W}{\partial\theta} \tag{5}$$

Abramzon and Borde (1980) have used the Peaceman-Rachford formulation of the alternating direction scheme to Eq. 5, which can be written as

$$\left(1 + \frac{\Delta t}{2r_i}f_i,k - \frac{p_1}{2}\delta_r^2 - \frac{q_1}{4}f_{i,k}\delta_{2r}\right)\left(1 - \frac{p_2}{2r_i^2}\delta_\theta^2 - \frac{q_2}{4}g_{i,k}\delta_{c2\theta}\right)W_{i,k}^{n+1} = \left(1 - \frac{\Delta t}{2r_i}f_{i,k} + \frac{p_1}{2}\delta_r^2 + \frac{q_1}{4}f_{i,k}\delta_{2r}\right)\left(1 + \frac{p_2}{2r_i^2}\delta_\theta^2 + \frac{q_2}{4}g_{i,k}\delta_{2\theta}\right)W_{i,k}^{n} \tag{6}$$

where $p_1 = \Delta t/\Delta r^2$, $q_1 = \Delta t/\Delta r$, $p_2 = \Delta t/\Delta\theta^2$, $q_2 = \Delta t/\Delta\theta$ and $\Delta t, \Delta r, \Delta\theta$ are step lengths in the time and space coordinates respectively. δ_r^2 and δ_{2r} denote the central difference approximations $\delta_{2r}W_{i,k} = W_{i+1,k} - W_{i-1,k}$, $\delta_r^2 W_{i,k} = W_{i+1,k} - 2W_{i,k} + W_{i-1,k}$ and similarly for δ_θ^2 and $\delta_{2\theta}$. The split form of Eq. 6 is

$$\left(1 + \frac{\Delta t}{2r_i}f_{i,k} - \frac{p_1}{2}\delta_r^2 - \frac{q_1}{4}f_{i,k}\delta_{2r}\right)W_{i,k}^{n+1/2} = \left(1 + \frac{p_2}{2r_i^2}\delta_\theta^2 + \frac{q_2}{4}g_{i,k}\delta_{2\theta}\right)W_{i,k}^{n} \tag{7}$$

$$\left(1 - \frac{p_2}{2r_i^2}\delta_\theta^2 - \frac{q_2}{4}g_{i,k}\delta_{2\theta}\right)W_{i,k}^{n+1} = \left(1 - \frac{\Delta t}{2r_i}f_{i,k} + \frac{p_1}{2}\delta_r^2 + \frac{q_1}{4}f_{i,k}\delta_{2r}\right)W_{i,k}^{n+1/2}. \tag{8}$$

These are the equations used by Abramzon and Borde (Eqs. 31 and 32 of the reference). The scheme (Eq. 6) is second order accurate. The schemes (Eqs. 7 and 8) are implicit and the system of equations produced are tridiagonal.

No stability analysis of the finite difference equations has been made by Abramzon and Borde. They have integrated Eqs. 7 and 8 using very small step lengths for Δt ranging from 10^{-6} to the start of computations to 10^{-2} at the final stage. We now discuss the stability of the scheme (Eq. 6). Using the Fourier method we find that the amplification factor of Eq. 6 is

$$G = \frac{(1-x_1) + i\,y_1}{(1+x_1) - i\,y_1}\cdot\frac{(1-x_2) + i\,y_2}{(1+x_2) - i\,y_2} \tag{9}$$

where

$$x_1 = \frac{f\Delta t}{2r_i} + 2p_1\sin^2(m\Delta r/2, \quad y_1 = \frac{q_1 f}{2}\sin(m\Delta r),$$

$$x_2 = \frac{2p_2}{r_i^2}\sin^2(m_1\Delta\theta/2), \qquad y_2 = \frac{q_2 g}{2}\sin(m_1\Delta\theta),$$

m, m_1 are the variables in the Fourier expansion and $f = f_{i,k}$, $g = g_{i,k}$. We find

$$|G|^2 = \frac{(1-x_1)^2 + y_1^2}{(1+x_1)^2 + y_1^2}\cdot\frac{(1-x_2)^2 + y_2^2}{(1+x_2)^2 + y_2^2} = F_1\cdot F_2$$

Let $F_1 = N_1/D_1$ and $F_2 = N_2/D_2$ where N and D stand for the numerator and denominator respectively. Write

$$F_1 = \frac{N_1}{N_1 + (D_1 - N_1)}, \; F_2 = \frac{N_2}{N_2 + (D_2 - N_2)}.$$

If $D_1 - N_1 \geqslant 0$ and $D_2 - N_2 \geqslant 0$, then $F_1 \leqslant 1$ and $F_2 \leqslant 1$. The scheme is then unconditionally stable, since $|G| \leqslant 1$. We find

$$D_1 - N_1 = 4x_1 = 4\left[2p_1\sin^2(m\Delta r/2) + \frac{\Delta t f}{2r_i}\right]$$

and

$$D_2 - N_2 = 4x_2 = \frac{8p_2}{r_i^2\sin^2(m_1\Delta\theta/2) \geqslant 0.}$$

When $\theta \geqslant \pi/2$, we have $f_{i,k} \geqslant 0$ and $D_1 - N_1 \geqslant 0$. Hence, scheme (Eq. 6) is unconditionally stable for $\theta \geqslant \pi/2$. The largest value of F_2 is unit. When $\theta < \pi/2$, we have $f_{i,k} < 0$ and $D_1 - N_1 = 2f\Delta t/r_i < 0$ for $m = 0$. This shows that at least for $m = 0$, $|G| > 1$ for $\theta < \pi/2$. This situation may also arise for some other values of m depending on the value of p_1. Hence, scheme (Eq. 6) is unstable for $\theta < \pi/2$. Scheme 6 can be used only for $\theta \geqslant \pi/2$ and because of unconditional stability one can use sufficiently large step length for Δt. For $\theta < \pi/2$, a stable difference approximation must be written, which the present author could not find. For $r > 1$, the difference scheme is of the form similar to Eq. 6. This scheme is unconditionally stable for $\theta \leqslant \pi/2$, while for $\theta > \pi/2$ it is unstable. Abramzon and Borde note that in the limiting case of $\mathrm{Pe} = 0$, their results are in excellent agreement with the exact solution of Cooper (1977). This is possible because when $\mathrm{Pe} = 0$, $f_{i,k} = 0$ and the scheme 6 is unconditionally stable for all θ. In this case, one can in fact use large step lengths for Δt. It is possible to construct unconditionally stable difference schemes in terms of the original variable $Z = W/r$. But then one need to satisfy the condition $Z(0,\theta,\tau) < \infty$ (Eq. 15 of reference).

NOTATION

K = viscosity ratio
r = dimensionless radial co-ordinate
Re = Reynold's number

Pe = Peclet number
W = unknown function, $W = rZ$
Z = dimensionless temperature, $Z = (T - T_\infty)/(T_o - T_\infty)$
θ = polar angle
Δt = step length in the time direction
Δr = step length in the radial direction
$\Delta\theta$ = angular step size
τ = dimensionless time.

LITERATURE CITED

Abramzon, B., and I. Borde, "Conjugate Unsteady Heat Transfer from a Droplet in Creeping Flow," *AIChE J.*, **26**, 536 (1980).

Cooper, F., "Heat Transfer from a Sphere to an Infinite Medium," *Int. J. Heat Mass Transfer*, **26**, 991 (1977).

Manuscript received April 27, 1981; revision received July 13, and accepted August 7, 1981.

BOOKS

Principles of Polymer Systems, Second Edition, Ferdinand Rodriguez, McGraw-Hill, 1982, 575 pages, $29.95, solutions manual available

On the general subject of polymers, this book is encyclopedic, with topics that would interest the polymer morphologist, chemist, and physicist, and rheologist, the manufacturer, and the design engineer. It therefore is very useful to the chemical engineer who wants exposure to a variety of polymer topics.

The first edition of this book, published twelve years ago, has proven extremely popular as a textbook and also as an initial reference book. This second edition is even better. But if you own the first edition, should you purchase the second? Many things remain the same, including chapter headings and subheadings and the total number of pages. But because of type compression and a greater number of lines per page, there are roughly 25% more words in the new edition. This word increase does not result from any chapter addition but rather appears to be spread throughout the book. It is clear that Rodriguez has done a careful job with this new revision. The lists of general references at the conclusion of each chapter have been updated and, more importantly, expanded fivefold. And most educators will appreciate the slight increase in the number of problems and the complete shift to metric units.

For those who do not own the first edition, yes, you should purchase the second edition if you are interested in polymers. The book possesses several particularly attractive features: (1) extensive breadth, including fifteen chapters covering such topics as structure and morphology, polymerization reactions and processes, viscoelastic properties, ultimate properties, degradation characteristics, and fabrication processes, (2) excellent figures and quantitative problems, (3) lucid prose and process description, (4) laboratory exercises, and (5) excellent lists of "General References" on each chapter topic. The first four features are central to its primary intended function as a textbook for junior and senior-level students and the reference lists allow the practicing engineer to use this as an initial reference text.

Some readers might prefer a greater depth of treatment on particular topics but this would detract from the nice balance Rodriguez has struck on depth and breadth. If one were allowed only one book on polymers, this would be the one to have.

CHARLES B. WEINBERGER
Department of Chemical Engineering
Drexel University
Philadelphia, PA 19104

Nuclear Chemical Engineering, 2nd. Ed., Manson Benedict, Thomas Pigford and Hans Levi, McGraw-Hill Book Company, New York, (1981), 1008 pages, $37.95.

When the first edition of this text (by the first two authors) appeared almost a quarter century ago in 1957, it was a time of rapid technological change. The second edition is written at a time of mature and consolidated technology and is in the authors words, "an entirely new book, following the first edition only in its general outline." There are some sections of chapters which are almost unchanged from the first edition, e.g., distillation and cascade analysis. The size of the book has almost doubled from 573 to 983 pages, while the number of chapters has gone from 12 to 14. These are: 1. Chemical Engineering Aspects of Nuclear Power; 2. Nuclear Reactions; 3. Fuel Cycles for Nuclear Reactors; 4. Solvent Extraction of Metals; 5. Uranium; 6. Thorium; 7. Zirconium and Hafnium; 8. Properties of Irradiated Fuel and Other Reactor Materials; 9. Plutonium and Other Actinide Elements; 10. Fuel Reprocessing; 11. Radioactive Waste Management; 12. Stable Isotopes: Uses, Separation Methods and Separation Principles; 13. Separation of Isotopes of Hydrogen and Other Light Elements; and, 14. Uranium Isotope Separation. Chapters 1–4, 8, 10, and 14 basically update Chapter 1–3, 6–8, and 12; Chapters 5, 6, and 7 expand considerably Chapters 4 and 5; and, Chapters 12 and 13 update and reorganize Chapters 9, 10, and 11 of the first edition. Chapters 9 and 11 are new. Each chapter has its own nomenclature, references, and problems as previously, but the number of problems are fewer, indicating the encyclopedic reference rather than didactic nature of the text. The chapter on Nuclear Waste Management appears to be quite up-to-date. Some figures are retained from the first edition, but most are new or redrawn. The mixture of English, cgs and SI units is sometimes jarring with pressure in Torr *and* psia on the same page, neither of which are SI units. Considering the effort of producing such a book, a little extra effort could have gone into consistent units. Conversion tables are supplied and dual units are frequent, however.

Engineers and scientists working in this area of the nuclear industry will want to have this text as a reference tool since it comprehensively covers the technologies and their scientific, nuclear and chemical principles.

ROBERT W. LYCZKOWSKI
Argonne National Laboratory
Components Technology Division
9700 South Cass Avenue
Argonne, IL 60439

Heat Transfer Engineering, Hemisphere Publishing Corp. 1025 Vermont Ave., N.W., Washington, D.C.-20005, 19 W. 44th St., New York-10036. (An international quarterly journal founded in 1979). Annual subscriptions are $18 for individuals or $40 for Institutions.

The editorial organization includes Western Europe and USA equally, with